

Higgs boson mass corrections in the $\mu\nu$ SSM with effective potential methods

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Abstract

To solve the μ problem of the MSSM, the μ from ν Supersymmetric Standard Model ($\mu\nu$ SSM) introduces three singlet right-handed neutrino superfields $\hat{\nu}_i^c$, which lead to the mixing of the neutral components of the Higgs doublets with the sneutrinos, producing a relatively large CP-even neutral scalar mass matrix. In this work, we analytically diagonalize the CP-even neutral scalar mass matrix and analyze in detail how the mixing impacts the lightest Higgs boson mass. We also give an approximate expression for the lightest Higgs boson mass. Simultaneously, we consider the radiative corrections to the Higgs boson masses with effective potential methods.

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I. INTRODUCTION

Since the ATLAS and CMS Collaborations reported the significant discovery of a new neutral Higgs boson [1, 2], the Higgs boson mass is now precisely measured by [3]

$$m_h = 125.09 \pm 0.24 \text{ GeV}. \quad (1)$$

Therefore, the accurate Higgs boson mass will give most stringent constraints on parameter space for the standard model and its various extensions.

As a supersymmetric model, the “ μ from ν supersymmetric standard model” ($\mu\nu$ SSM) has the superpotential: [4–10]

$$W = \epsilon_{ab} \left(Y_{u_{ij}} \hat{H}_u^b \hat{Q}_i^a \hat{u}_j^c + Y_{d_{ij}} \hat{H}_d^a \hat{Q}_i^b \hat{d}_j^c + Y_{e_{ij}} \hat{H}_d^a \hat{L}_i^b \hat{e}_j^c + Y_{\nu_{ij}} \hat{H}_u^b \hat{L}_i^a \hat{\nu}_j^c \right) - \epsilon_{ab} \lambda_i \hat{\nu}_i^c \hat{H}_d^a \hat{H}_u^b + \frac{1}{3} \kappa_{ijk} \hat{\nu}_i^c \hat{\nu}_j^c \hat{\nu}_k^c, \quad (2)$$

where $\hat{H}_u^T = (\hat{H}_u^+, \hat{H}_u^0)$, $\hat{H}_d^T = (\hat{H}_d^0, \hat{H}_d^-)$, $\hat{Q}_i^T = (\hat{u}_i, \hat{d}_i)$, $\hat{L}_i^T = (\hat{\nu}_i, \hat{e}_i)$ are $SU(2)$ doublet superfields, and $Y_{u,d,e,\nu}$, λ , and κ are dimensionless matrices, a vector, and a totally symmetric tensor, respectively. $a, b = 1, 2$ are $SU(2)$ indices with antisymmetric tensor $\epsilon_{12} = 1$, and $i, j, k = 1, 2, 3$ are generation indices. The summation convention is implied on repeated indices in this paper. Besides the superfields of the MSSM [11–15], the $\mu\nu$ SSM introduces three singlet right-handed neutrino superfields $\hat{\nu}_i^c$ to solve the μ problem [16] of the MSSM. Once the electroweak symmetry is broken (EWSB), the effective μ term $-\epsilon_{ab}\mu\hat{H}_d^a\hat{H}_u^b$ is generated spontaneously through right-handed sneutrino vacuum expectation values (VEVs), $\mu = \lambda_i \langle \hat{\nu}_i^c \rangle$. Additionally, three tiny neutrino masses can be generated at the tree level through a TeV scale seesaw mechanism [4–9, 17–23].

In the $\mu\nu$ SSM, the left- and right-handed sneutrino VEVs lead to the mixing of the neutral components of the Higgs doublets with the sneutrinos producing an 8×8 CP-even neutral scalar mass matrix, which can be seen in Refs. [5–7]. Therefore, the mixing would affect the lightest Higgs boson mass. In this work, we analytically diagonalize the CP-even neutral scalar mass matrix, which would be conducive to the follow-up study on the Higgs sector. In the meantime, we consider the Higgs boson mass corrections with effective potential methods. We also give an approximate expression for the lightest Higgs boson

mass. In numerical analysis, we will analyze how the mixing affects the lightest Higgs boson mass.

Our presentation is organized as follows. In Sec. II, we briefly summarize the Higgs sector of the $\mu\nu$ SSM, including the Higgs boson mass corrections. We present the diagonalization of the neutral scalar mass matrix analytically in Sec. III. The numerical analyses are given in Sec. IV, and Sec. V provides a summary. The tedious formulas are collected in the Appendixes.

II. THE HIGGS SECTOR

The Higgs sector of the $\mu\nu$ SSM contains the usual two Higgs doublets with the left- and right-handed sneutrinos: $\hat{H}_d^T = (\hat{H}_d^0, \hat{H}_d^-)$, $\hat{H}_u^T = (\hat{H}_u^+, \hat{H}_u^0)$, $\hat{\nu}_i$ and $\hat{\nu}_i^c$. Once EWSB, the neutral scalars have the VEVs:

$$\langle H_d^0 \rangle = v_d, \quad \langle H_u^0 \rangle = v_u, \quad \langle \tilde{\nu}_i \rangle = v_{\nu_i}, \quad \langle \tilde{\nu}_i^c \rangle = v_{\nu_i^c}. \quad (3)$$

One can define the neutral scalars as

$$\begin{aligned} H_d^0 &= \frac{1}{\sqrt{2}}(h_d + iP_d) + v_d, & \tilde{\nu}_i &= \frac{1}{\sqrt{2}}((\tilde{\nu}_i)^{\Re} + i(\tilde{\nu}_i)^{\Im}) + v_{\nu_i}, \\ H_u^0 &= \frac{1}{\sqrt{2}}(h_u + iP_u) + v_u, & \tilde{\nu}_i^c &= \frac{1}{\sqrt{2}}((\tilde{\nu}_i^c)^{\Re} + i(\tilde{\nu}_i^c)^{\Im}) + v_{\nu_i^c}, \end{aligned} \quad (4)$$

Considering that the neutrino oscillation data constrain neutrino Yukawa couplings $Y_{\nu_i} \sim \mathcal{O}(10^{-7})$ and left-handed sneutrino VEVs $v_{\nu_i} \sim \mathcal{O}(10^{-4}\text{GeV})$ [4–7, 17–22], in the following we could reasonably neglect the small terms including Y_ν or v_{ν_i} in the Higgs sector. Then, the superpotential in Eq. (2) approximately leads to the tree-level neutral scalar (Higgs) potential:

$$V^0 = V_F + V_D + V_{soft}, \quad (5)$$

with

$$\begin{aligned} V_F &= \lambda_i \lambda_i^* H_d^0 H_d^{0*} H_u^0 H_u^{0*} + \lambda_i \lambda_j^* \tilde{\nu}_i^c \tilde{\nu}_j^{c*} (H_d^0 H_d^{0*} + H_u^0 H_u^{0*}) \\ &\quad + \kappa_{ijk} \kappa_{ljm}^* \tilde{\nu}_i^c \tilde{\nu}_k^c \tilde{\nu}_l^{c*} \tilde{\nu}_m^{c*} - (\kappa_{ijk} \lambda_j^* \tilde{\nu}_i^c \tilde{\nu}_k^c H_d^0 H_u^{0*} + \text{H.c.}), \end{aligned} \quad (6)$$

$$V_D = \frac{G^2}{8}(\tilde{\nu}_i \tilde{\nu}_i^* + H_d^0 H_d^{0*} - H_u^0 H_u^{0*})^2, \quad (7)$$

$$V_{soft} = m_{H_d}^2 H_d^0 H_d^{0*} + m_{H_u}^2 H_u^0 H_u^{0*} + m_{\tilde{L}_{ij}}^2 \tilde{\nu}_i \tilde{\nu}_j^* + m_{\tilde{\nu}_{ij}^c}^2 \tilde{\nu}_i^c \tilde{\nu}_j^{c*} \\ - \left((A_\lambda \lambda)_i \nu_i^c H_d^0 H_u^0 - \frac{1}{3} (A_\kappa \kappa)_{ijk} \tilde{\nu}_i^c \tilde{\nu}_j^c \tilde{\nu}_k^c + \text{H.c.} \right), \quad (8)$$

where $G^2 = g_1^2 + g_2^2$ and $g_1 c_W = g_2 s_W = e$, V_F and V_D are the usual F and D terms derived from the superpotential, and V_{soft} denotes the soft supersymmetry breaking terms. For simplicity, we will assume that all parameters in the potential are real in the following.

With effective potential methods [24–39], the one-loop effective potential can be given by

$$V^1 = \frac{1}{32\pi^2} \left\{ \sum_{\tilde{f}} N_f m_{\tilde{f}}^4 \left(\log \frac{m_{\tilde{f}}^2}{Q^2} - \frac{3}{2} \right) - 2 \sum_{f=t,b,\tau} N_f m_f^4 \left(\log \frac{m_f^2}{Q^2} - \frac{3}{2} \right) \right\}, \quad (9)$$

where, Q denotes the renormalization scale, $N_t = N_b = 3$ and $N_\tau = 1$, $\tilde{f} = \tilde{t}_{1,2}, \tilde{b}_{1,2}, \tilde{\tau}_{1,2}$. The masses of the third fermions $f = t, b, \tau$ and corresponding supersymmetric partners $\tilde{f} = \tilde{t}_{1,2}, \tilde{b}_{1,2}, \tilde{\tau}_{1,2}$ in the $\mu\nu$ SSM are collected in Appendix A. Including the one-loop effective potential, the Higgs potential is written as

$$V = V^0 + V^1. \quad (10)$$

Through the Higgs potential, we will calculate the minimization conditions of the potential and the Higgs masses in the following.

Minimizing the Higgs potential, we can obtain the minimization conditions of the potential, linking the soft mass parameters to the VEVs of the neutral scalar fields:

$$m_{H_d}^2 = -\Delta T_{H_d} + ((A_\lambda \lambda)_i \nu_{\nu_i^c} + \lambda_j \kappa_{ijk} \nu_{\nu_i^c} \nu_{\nu_k^c}) \tan \beta \\ - (\lambda_i \lambda_j \nu_{\nu_i^c} \nu_{\nu_j^c} + \lambda_i \lambda_i v_u^2) + \frac{G^2}{4} (v_u^2 - v_d^2), \quad (11)$$

$$m_{H_u}^2 = -\Delta T_{H_u} + ((A_\lambda \lambda)_i \nu_{\nu_i^c} + \lambda_j \kappa_{ijk} \nu_{\nu_i^c} \nu_{\nu_k^c}) \cot \beta \\ - (\lambda_i \lambda_j \nu_{\nu_i^c} \nu_{\nu_j^c} + \lambda_i \lambda_i v_d^2) + \frac{G^2}{4} (v_d^2 - v_u^2), \quad (12)$$

$$m_{\tilde{\nu}_{ij}^c}^2 \nu_{\nu_j^c} = -\Delta T_{\tilde{\nu}_{ij}^c} \nu_{\nu_j^c} + (A_\lambda \lambda)_i \nu_d \nu_u - (A_\kappa \kappa)_{ijk} \nu_{\nu_j^c} \nu_{\nu_k^c} + 2\lambda_j \kappa_{ijk} \nu_{\nu_k^c} \nu_d \nu_u \\ - 2\kappa_{lim} \kappa_{ljk} \nu_{\nu_m^c} \nu_{\nu_j^c} \nu_{\nu_k^c} - \lambda_i \lambda_j \nu_{\nu_j^c} (v_d^2 + v_u^2), \quad (i = 1, 2, 3) \quad (13)$$

where, as usual, $\tan \beta = v_u/v_d$. ΔT_{H_d} , ΔT_{H_u} , and $\Delta T_{\tilde{\nu}_{ij}^c} \nu_{\nu_j^c}$ come from one-loop corrections to the minimization conditions, which are taken in Appendix B. Here, neglecting the small

terms including Y_ν or v_{ν_i} in the Higgs sector, we do not give the minimization conditions of the potential about the left-handed sneutrino VEVs, which can be used to constrain v_{ν_i} [17, 22].

From the Higgs potential, one can derive the 8×8 mass matrices for the CP-even neutral scalars $S'^T = (h_d, h_u, (\tilde{\nu}_i^c)^{\Re}, (\tilde{\nu}_i)^{\Re})$ and the CP-odd neutral scalars $P'^T = (P_d, P_u, (\tilde{\nu}_i^c)^{\Im}, (\tilde{\nu}_i)^{\Im})$ in the unrotated basis. Ignoring the small terms including Y_ν or v_{ν_i} , the 5×5 mass submatrix for Higgs doublets and right-handed sneutrinos is basically decoupled from the 3×3 left-handed sneutrinos mass submatrix. The 3×3 left-handed sneutrino mass submatrix is $(m_{L_{ij}}^2 + \frac{G^2}{4}(v_d^2 - v_u^2)\delta_{ij})_{3 \times 3}$, which is dominated by the soft mass $m_{L_{ij}}^2$. Through the Higgs potential, the 5×5 mass submatrix for Higgs doublets and right-handed sneutrinos in the CP-even sector can be derived as

$$M_S^2 = \begin{pmatrix} M_H^2 & M_X^2 \\ (M_X^2)^T & M_R^2 \end{pmatrix}, \quad (14)$$

where M_H^2 denotes the 2×2 mass submatrix for Higgs doublets, M_R^2 is the 3×3 mass submatrix for right-handed sneutrinos and M_X^2 represents the 2×3 mass submatrix for the mixing of Higgs doublets and right-handed sneutrinos.

In detail, the 2×2 mass submatrix M_H^2 can be written by

$$M_H^2 = \begin{pmatrix} M_{h_d h_d}^2 + \Delta_{11} & M_{h_d h_u}^2 + \Delta_{12} \\ M_{h_d h_u}^2 + \Delta_{12} & M_{h_u h_u}^2 + \Delta_{22} \end{pmatrix}, \quad (15)$$

with the tree-level contributions as

$$M_{h_d h_u}^2 = -[m_A^2 + (1 - 4\lambda_i \lambda_i s_W^2 c_W^2 / e^2)m_Z^2] \sin \beta \cos \beta, \quad (16)$$

$$M_{h_d h_d}^2 = m_A^2 \sin^2 \beta + m_Z^2 \cos^2 \beta, \quad (17)$$

$$M_{h_u h_u}^2 = m_A^2 \cos^2 \beta + m_Z^2 \sin^2 \beta, \quad (18)$$

and the neutral pseudoscalar mass squared as

$$m_A^2 \simeq \frac{2}{\sin 2\beta} [(A_\lambda \lambda)_i v_{\nu_i^c} + \lambda_k \kappa_{ijk} v_{\nu_i^c} v_{\nu_j^c}]. \quad (19)$$

Comparing with the MSSM, $M_{h_d h_u}^2$ has an additional term $(4\lambda_i \lambda_i s_W^2 c_W^2 / e^2)m_Z^2 \sin \beta \cos \beta$, which can give a new contribution to the lightest Higgs boson mass. The radiative corrections

Δ_{11} , Δ_{12} , and Δ_{22} from the third fermions $f = t, b, \tau$ and their superpartners can be found in Ref. [9], which agree with the results of the MSSM [24–37]. Here, the radiative corrections from the top quark and its superpartners include the two-loop leading-log effects, which can obviously affect the mass of the lightest Higgs boson.

Furthermore, the 2×3 mixing mass submatrix M_X^2 is

$$M_X^2 = \begin{pmatrix} (M_{hd(\tilde{\nu}_i^c)}^2 + \Delta_{1(2+i)})_{1 \times 3} \\ (M_{hu(\tilde{\nu}_i^c)}^2 + \Delta_{2(2+i)})_{1 \times 3} \end{pmatrix}, \quad (20)$$

where

$$M_{hd(\tilde{\nu}_i^c)}^2 = [2\lambda_i \lambda_j v_{\nu_j^c} \cot \beta - ((A_\lambda \lambda)_i + 2\lambda_k \kappa_{ijk} v_{\nu_j^c})] v_u, \quad (21)$$

$$M_{hu(\tilde{\nu}_i^c)}^2 = [2\lambda_i \lambda_j v_{\nu_j^c} \tan \beta - ((A_\lambda \lambda)_i + 2\lambda_k \kappa_{ijk} v_{\nu_j^c})] v_d, \quad (22)$$

and the radiative corrections from the third fermions $f = t, b, \tau$ and their superpartners are

$$\Delta_{1(2+i)} = \lambda_i v_u \Delta_{1R}, \quad \Delta_{2(2+i)} = \lambda_i v_d \Delta_{2R}, \quad (23)$$

$$\begin{aligned} \Delta_{1R} = \frac{G_F}{2\sqrt{2}\pi^2} \bigg\{ & \frac{3m_t^4}{\sin^2 \beta} \frac{\mu(A_t - \mu \cot \beta)^2}{\tan \beta (m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \\ & + \frac{3m_b^4}{\cos^2 \beta} \frac{(-A_b + \mu \tan \beta)}{(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)} \left[\log \frac{m_{\tilde{b}_1}^2}{m_{\tilde{b}_2}^2} + \frac{A_b(A_b - \mu \tan \beta)}{(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)} g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) \right] \\ & + \frac{m_\tau^4}{\cos^2 \beta} \frac{(-A_\tau + \mu \tan \beta)}{(m_{\tilde{\tau}_1}^2 - m_{\tilde{\tau}_2}^2)} \left[\log \frac{m_{\tilde{\tau}_1}^2}{m_{\tilde{\tau}_2}^2} + \frac{A_\tau(A_\tau - \mu \tan \beta)}{(m_{\tilde{\tau}_1}^2 - m_{\tilde{\tau}_2}^2)} g(m_{\tilde{\tau}_1}^2, m_{\tilde{\tau}_2}^2) \right] \bigg\}, \quad (24) \end{aligned}$$

$$\begin{aligned} \Delta_{2R} = \frac{G_F}{2\sqrt{2}\pi^2} \bigg\{ & \frac{3m_t^4}{\sin^2 \beta} \frac{(-A_t + \mu \cot \beta)}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)} \left[\log \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} + \frac{A_t(A_t - \mu \cot \beta)}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right] \\ & + \frac{3m_b^4}{\cos^2 \beta} \frac{\mu(A_b - \mu \tan \beta)^2}{\cot \beta (m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)^2} g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) \\ & + \frac{m_\tau^4}{\cos^2 \beta} \frac{\mu(A_\tau - \mu \tan \beta)^2}{\cot \beta (m_{\tilde{\tau}_1}^2 - m_{\tilde{\tau}_2}^2)^2} g(m_{\tilde{\tau}_1}^2, m_{\tilde{\tau}_2}^2) \bigg\}, \quad (25) \end{aligned}$$

with $\mu = \lambda_i v_{\nu_i^c}$, $g(m_1^2, m_2^2) = 2 - \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2} \log \frac{m_1^2}{m_2^2}$. Here, we can know that the radiative corrections to the mixing are proportional to the parameters λ_i .

Similarly, one can derive the 3×3 mass submatrix for the right-handed sneutrinos:

$$M_R^2 = \left(M_{(\tilde{\nu}_i^c)\mathfrak{R}(\tilde{\nu}_j^c)\mathfrak{R}}^2 + \Delta_{(2+i)(2+j)} \right)_{3 \times 3}, \quad (26)$$

with

$$M_{(\tilde{\nu}_i^c)\Re(\tilde{\nu}_j^c)\Re}^2 = m_{\tilde{\nu}_{ij}^c}^2 + 2(A_\kappa \kappa)_{ijk} v_{\nu_k^c} - 2\lambda_k \kappa_{ijk} v_d v_u + \lambda_i \lambda_j (v_d^2 + v_u^2) \\ + (2\kappa_{ijk} \kappa_{lmk} + 4\kappa_{ilk} \kappa_{jmk}) v_{\nu_l^c} v_{\nu_m^c}, \quad (27)$$

and the corrections from the third fermions and their superpartners are

$$\Delta_{(2+i)(2+j)} = \lambda_i \lambda_j \Delta_{RR}, \quad (28)$$

$$\Delta_{RR} = \frac{G_F}{2\sqrt{2}\pi^2} \left\{ \frac{3m_t^4}{\sin^2 \beta} \frac{v_d^2 (A_t - \mu \cot \beta)^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right. \\ + \frac{3m_b^4}{\cos^2 \beta} \frac{v_u^2 (A_b - \mu \tan \beta)^2}{(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)^2} g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) \\ \left. + \frac{m_\tau^4}{\cos^2 \beta} \frac{v_u^2 (A_\tau - \mu \tan \beta)^2}{(m_{\tilde{\tau}_1}^2 - m_{\tilde{\tau}_2}^2)^2} g(m_{\tilde{\tau}_1}^2, m_{\tilde{\tau}_2}^2) \right\}. \quad (29)$$

Here, the radiative corrections to the mass submatrix for right-handed sneutrinos are proportional to $\lambda_i \lambda_j$.

III. DIAGONALIZATION OF THE MASS MATRIX

The mass squared matrix M_H^2 which contains the radiative corrections can be diagonalized as

$$U_H^T M_H^2 U_H = \text{diag}(m_{H_1}^2, m_{H_2}^2), \quad (30)$$

by the 2×2 unitary matrix U_H ,

$$U_H = \begin{pmatrix} -\sin \alpha & \cos \alpha \\ \cos \alpha & \sin \alpha \end{pmatrix}. \quad (31)$$

Here, the neutral doubletlike Higgs mass squared eigenvalues $m_{H_{1,2}}^2$ can be derived,

$$m_{H_{1,2}}^2 = \frac{1}{2} \left[\text{Tr} M_H^2 \mp \sqrt{(\text{Tr} M_H^2)^2 - 4 \text{Det} M_H^2} \right], \quad (32)$$

where $\text{Tr} M_H^2 = M_{H11}^2 + M_{H22}^2$, $\text{Det} M_H^2 = M_{H11}^2 M_{H22}^2 - (M_{H12}^2)^2$. The mixing angle α can be determined by [32]

$$\sin 2\alpha = \frac{2M_{H12}^2}{\sqrt{(\text{Tr} M_H^2)^2 - 4 \text{Det} M_H^2}},$$

$$\cos 2\alpha = \frac{M_{H11}^2 - M_{H22}^2}{\sqrt{(\text{Tr} M_H^2)^2 - 4\text{Det} M_H^2}}, \quad (33)$$

which reduce to $-\sin 2\beta$ and $-\cos 2\beta$, respectively, in the large m_A limit. The convention is that $\pi/4 \leq \beta < \pi/2$ for $\tan \beta \geq 1$, while $-\pi/2 < \alpha < 0$. In the large m_A limit, $\alpha = -\pi/2 + \beta$.

In the large m_A limit, the light neutral doubletlike Higgs mass is approximately given as

$$m_{H_1}^2 \simeq m_Z^2 \cos^2 2\beta + \frac{2\lambda_i \lambda_i s_W^2 c_W^2}{e^2} m_Z^2 \sin^2 2\beta + \Delta m_{H_1}^2. \quad (34)$$

Comparing with the MSSM, the $\mu\nu$ SSM gets an additional term $\frac{2\lambda_i \lambda_i s_W^2 c_W^2}{e^2} m_Z^2 \sin^2 2\beta$ [5]. Here, the radiative corrections $\Delta m_{H_1}^2$ can be computed more precisely by some public tools, for example, FeynHiggs [40–47], SOFTSUSY [48–50], SPheno [51, 52], and so on. In the following numerical section, we will use the FeynHiggs-2.13.0 to calculate the radiative corrections for the Higgs boson mass about the MSSM part.

To further deal with the mass submatrix M_R^2 and M_X^2 , in the following we choose the usual minimal scenario for the parameter space:

$$\begin{aligned} \lambda_i &= \lambda, \quad (A_\lambda \lambda)_i = A_\lambda \lambda, \quad v_{\nu_i^c} = v_{\nu^c}, \\ \kappa_{ijk} &= \kappa \delta_{ij} \delta_{jk}, \quad (A_\kappa \kappa)_{ijk} = A_\kappa \kappa \delta_{ij} \delta_{jk}, \quad m_{\tilde{\nu}_{ij}^c}^2 = m_{\tilde{\nu}_i^c}^2 \delta_{ij}, \end{aligned} \quad (35)$$

Then, the 3×3 mass submatrix for CP-even right-handed sneutrinos can be simplified as

$$M_R^2 = \begin{pmatrix} X_R & y_R & y_R \\ y_R & X_R & y_R \\ y_R & y_R & X_R \end{pmatrix}, \quad (36)$$

with

$$X_R = (A_\kappa + 4\kappa v_{\nu^c}) \kappa v_{\nu^c} + A_\lambda \lambda v_d v_u / v_{\nu^c} + \lambda^2 \Delta_{RR}, \quad (37)$$

$$y_R = \lambda^2 (v^2 + \Delta_{RR}), \quad (38)$$

where $v^2 = v_d^2 + v_u^2$. Here the radiative corrections keep the dominating contributions which are proportional to m_f^4 ($f = t, b, \tau$). Through the 3×3 unitary matrix U_R ,

$$U_R = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{pmatrix}, \quad (39)$$

the mass squared matrix M_R^2 can be diagonalized as

$$U_R^T M_R^2 U_R = \text{diag}(m_{R_1}^2, m_{R_2}^2, m_{R_3}^2), \quad (40)$$

with

$$m_{R_1}^2 = X_R + 2y_R = (A_\kappa + 4\kappa v_{\nu^c})\kappa v_{\nu^c} + A_\lambda \lambda v_d v_u / v_{\nu^c} + \lambda^2(2v^2 + 3\Delta_{RR}), \quad (41)$$

$$m_{R_2}^2 = m_{R_3}^2 = X_R - y_R = (A_\kappa + 4\kappa v_{\nu^c})\kappa v_{\nu^c} + A_\lambda \lambda v_d v_u / v_{\nu^c} - \lambda^2 v^2. \quad (42)$$

The radiative corrections are proportional to λ^2 , which will be tamped down as $\lambda \sim \mathcal{O}(0.1)$. Then the masses squared of the CP-even right-handed sneutrinos can be approximated by

$$m_{S_R}^2 \approx m_{R_1}^2 \approx m_{R_2}^2 = m_{R_3}^2 \approx (A_\kappa + 4\kappa v_{\nu^c})\kappa v_{\nu^c} + A_\lambda \lambda v_d v_u / v_{\nu^c}. \quad (43)$$

Due to $v_{\nu^c} \gg v_{u,d}$, the main contribution to the mass squared is the first term as κ is large. Additionally, the masses squared of the CP-odd right-handed sneutrinos $m_{P_R}^2$ can be approximated as

$$m_{P_R}^2 \approx -3A_\kappa \kappa v_{\nu^c} + (4\kappa + A_\lambda / v_{\nu^c})\lambda v_d v_u, \quad (44)$$

where the first term is the leading contribution. Therefore, one can use the approximate relation,

$$-4\kappa v_{\nu^c} \lesssim A_\kappa \lesssim 0, \quad (45)$$

to avoid the tachyons.

In the minimal scenario for the parameter space presented in Eq. (35), the 2×3 mixing mass submatrix M_X^2 is simplified as

$$M_X^2 = \begin{pmatrix} M_{X_1}^2 & M_{X_1}^2 & M_{X_1}^2 \\ M_{X_2}^2 & M_{X_2}^2 & M_{X_2}^2 \end{pmatrix}, \quad (46)$$

where

$$M_{X_1}^2 = \lambda v \sin \beta [2v_{\nu^c}(3\lambda \cot \beta - \kappa) - A_\lambda + \Delta_{1R}], \quad (47)$$

$$M_{X_2}^2 = \lambda v \cos \beta [2v_{\nu^c}(3\lambda \tan \beta - \kappa) - A_\lambda + \Delta_{2R}]. \quad (48)$$

Then, we do the calculation:

$$\begin{pmatrix} U_H^T & 0 \\ 0 & U_R^T \end{pmatrix} \begin{pmatrix} M_H^2 & M_X^2 \\ (M_X^2)^T & M_R^2 \end{pmatrix} \begin{pmatrix} U_H & 0 \\ 0 & U_R \end{pmatrix} = \mathcal{H} \oplus \begin{pmatrix} m_{R_2}^2 & 0 \\ 0 & m_{R_3}^2 \end{pmatrix}, \quad (49)$$

with

$$\mathcal{H} = \begin{pmatrix} m_{H_1}^2 & 0 & A_{X_1}^2 \\ 0 & m_{H_2}^2 & A_{X_2}^2 \\ A_{X_1}^2 & A_{X_2}^2 & m_{R_1}^2 \end{pmatrix}, \quad (50)$$

where

$$A_{X_1}^2 = \sqrt{3}(-M_{X_1}^2 \sin \alpha + M_{X_2}^2 \cos \alpha), \quad (51)$$

$$A_{X_2}^2 = \sqrt{3}(M_{X_1}^2 \cos \alpha + M_{X_2}^2 \sin \alpha). \quad (52)$$

In the large m_A limit, $\alpha = -\pi/2 + \beta$. Then, one can have the following approximate expressions:

$$A_{X_1}^2 \simeq \sqrt{3}\lambda v \sin 2\beta \left[2v_{\nu^c} \left(\frac{3\lambda}{\sin 2\beta} - \kappa \right) - A_\lambda + \frac{1}{2}(\Delta_{1R} + \Delta_{2R}) \right], \quad (53)$$

$$A_{X_2}^2 \simeq \sqrt{3}\lambda v \left[(2\kappa v_{\nu^c} + A_\lambda) \cos 2\beta + \Delta_{1R} \sin^2 \beta - \Delta_{2R} \cos^2 \beta \right]. \quad (54)$$

If $A_{X_1}^2 = 0$, the mixing of Higgs doublets and right-handed sneutrinos will not affect the lightest Higgs boson mass [5]; namely, one can adopt the relation

$$A_\lambda = 2v_{\nu^c} \left(\frac{3\lambda}{\sin 2\beta} - \kappa \right) + \frac{1}{2}(\Delta_{1R} + \Delta_{2R}), \quad (55)$$

which is analogous to the NMSSM [53, 54]. To relax the conditions, if A_λ is around the value in Eq. (55), the contribution to the lightest Higgs boson mass from the mixing could also be neglected approximately. In the case $A_{X_1}^2 \approx 0$, the mass of the lightest Higgs boson is just m_{H_1} , which shows, approximately, in Eq. (34).

If $A_{X_1}^2 \neq 0$, we need to diagonalize the 3×3 mass matrix \mathcal{H} further:

$$U_X^T \mathcal{H} U_X = \text{diag}(m_h^2, m_H^2, m_{S_3}^2), \quad (56)$$

where the eigenvalues $m_h^2, m_H^2, m_{S_3}^2$ and the unitary matrix U_X can be concretely seen in Appendix C. Then, the lightest Higgs boson mass is exactly m_h^2 . In the large m_A limit,

$m_{H_2} \simeq m_A$, one can have the lightest Higgs boson mass squared approximately as

$$m_h^2 \simeq \frac{1}{2} \left\{ m_{H_1}^2 + m_{R_1}^2 - \frac{(A_{X_2}^2)^2}{m_{H_2}^2} - \sqrt{\left[m_{R_1}^2 - m_{H_1}^2 - \frac{(A_{X_2}^2)^2}{m_{H_2}^2} \right]^2 + 4(A_{X_1}^2)^2} \right\}. \quad (57)$$

The approximate expression works well, which can be easily checked in the numerical calculation. When m_{H_2} and m_{R_1} are all large, Eq. (57) could be approximated by

$$m_h^2 \approx m_{H_1}^2 - \frac{(A_{X_1}^2)^2}{m_{R_1}^2} = m_{H_1}^2 \left[1 - \frac{(A_{X_1}^2)^2}{m_{R_1}^2 m_{H_1}^2} \right]. \quad (58)$$

In the numerical analysis, we can define the quantity

$$\xi_h = \frac{(A_{X_1}^2)^2}{m_{R_1}^2 m_{H_1}^2} \quad (59)$$

to analyze how the mixing affects the mass of the lightest Higgs boson.

One can diagonalize the 5×5 mass submatrix for Higgs doublets and right-handed sneutrinos in the CP-even sector:

$$R_S^T M_S^2 R_S = \text{diag}(m_{S_1}^2, m_{S_2}^2, m_{S_3}^2, m_{S_4}^2, m_{S_5}^2), \quad (60)$$

with $m_{S_1} = m_h$, $m_{S_2} = m_H$, $m_{S_4} = m_{R_2} = m_{S_5} = m_{R_3}$, and the 5×5 unitary matrix R_S

$$R_S = \begin{pmatrix} U_H & 0 \\ 0 & U_R \end{pmatrix} \begin{pmatrix} U_X & 0 \\ 0 & I_{2 \times 2} \end{pmatrix}, \quad (61)$$

where $I_{2 \times 2}$ denotes the 2×2 unit matrix.

IV. NUMERICAL ANALYSIS

In this section, we will do the numerical analysis for the masses of the Higgs bosons. First, we choose the values of the parameter space. For the relevant parameters in the SM, we choose

$$\begin{aligned} \alpha_s(m_Z) &= 0.118, & m_Z &= 91.188 \text{ GeV}, & m_W &= 80.385 \text{ GeV}, \\ m_t &= 173.2 \text{ GeV}, & m_b &= 4.66 \text{ GeV}, & m_\tau &= 1.777 \text{ GeV}. \end{aligned} \quad (62)$$

The other SM parameters can be seen in Ref. [55] from the Particle Data Group. Here, we choose a suitable $A_\kappa = -500 \text{ GeV}$ to avoid the tachyons easily, through Eq. (45).

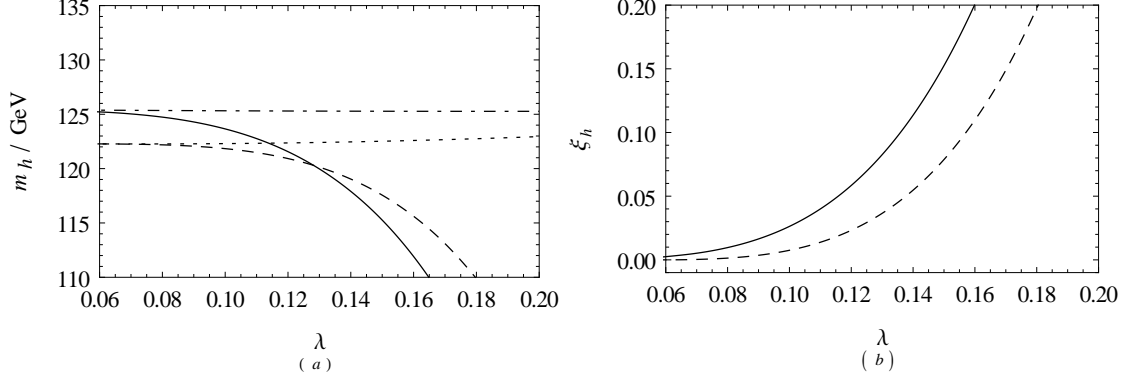


FIG. 1: (a) m_h varies with λ , the solid line and dash-dot line denote m_h and m_{H_1} as $\tan \beta = 20$, the dash line and dot line denote m_h and m_{H_1} as $\tan \beta = 6$. (b) ξ_h varies with λ , the solid line and dash line represent as $\tan \beta = 20$ and $\tan \beta = 6$, respectively. When $\kappa = 0.4$, $A_\lambda = 500$ GeV and $v_{\nu^c} = 2$ TeV.

Considering the direct search for supersymmetric particles [55], we could reasonably choose $M_2 = 2M_1 = 800$ GeV, $M_3 = 2$ TeV, $m_{\tilde{Q}_3} = m_{\tilde{U}_3} = m_{\tilde{D}_3} = 2$ TeV, $m_{\tilde{L}_3} = m_{\tilde{E}_3} = 1$ TeV, $A_b = A_\tau = 1$ TeV, and $A_t = 2.5$ TeV for simplicity. As key parameters, $m_{\tilde{Q}_3}$, $m_{\tilde{U}_3}$, A_t and the gaugino mass parameters affect the radiative corrections to the lightest Higgs mass. Therefore, one can take the proper values for $m_{\tilde{Q}_3}$, $m_{\tilde{U}_3}$, A_t and the gaugino mass parameters to keep the lightest Higgs mass around 125 GeV.

In the following, we will analyze how the mixing of Higgs doublets and right-handed sneutrinos affects the lightest Higgs boson mass. Through $A_{X_1}^2$ in Eq. (53), one knows that the parameters which affect the lightest Higgs boson mass from the mixing will be λ , $\tan \beta$, κ , A_λ , and v_{ν^c} . Here, we specify that the parameter $\mu = 3\lambda v_{\nu^c}$, which is dominated by the parameters λ and v_{ν^c} .

When $\kappa = 0.4$, $A_\lambda = 500$ GeV, and $v_{\nu^c} = 2$ TeV, we plot the lightest Higgs boson mass m_h , varying with the parameter λ in Fig. 1(a), where the solid line and dash-dot line denote m_h and m_{H_1} as $\tan \beta = 20$, the dash line and dot line denote m_h and m_{H_1} as $\tan \beta = 6$, respectively. The mass m_{H_1} denotes the lightest Higgs boson mass if we do not consider the mixing of Higgs doublets and right-handed sneutrinos, and the mass m_h is exactly the lightest Higgs boson mass considering the mixing. The numerical results indicate that the mixing could have significant effects on the lightest Higgs boson mass, as the parameter λ is

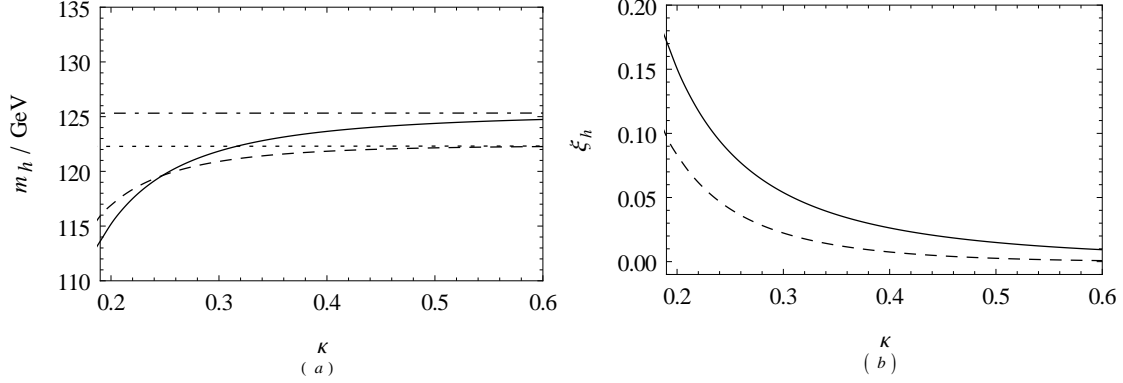


FIG. 2: (a) m_h varies with κ , the solid line and dash-dot line denote m_h and m_{H_1} as $\tan\beta = 20$, the dash line and dot line denote m_h and m_{H_1} as $\tan\beta = 6$. (b) ξ_h varies with κ , the solid line and dash line represent as $\tan\beta = 20$ and $\tan\beta = 6$, respectively. When $\lambda = 0.1$, $A_\lambda = 500$ GeV and $v_{\nu^c} = 2$ TeV.

large. With an increase of λ , the lightest Higgs boson mass m_h drops down quickly, which deviates from the mass m_{H_1} . For large $\tan\beta$, the lightest Higgs boson mass m_h decreases more quickly with increasing λ .

To see the reason more clearly, we also plot the quantity ξ_h , varying with λ in Fig. 1(b), where the solid line and dash line, respectively, represent as $\tan\beta = 20$ and $\tan\beta = 6$. The quantity ξ_h is defined in Eq. (59) to quantify the effect on the lightest Higgs boson mass from the mixing of Higgs doublets and right-handed sneutrinos. The figure shows that ξ_h increases quickly with an increase of λ , and ξ_h for large $\tan\beta$ is larger than it is for small $\tan\beta$. When λ is small, ξ_h is also small, and then m_h is close to m_{H_1} because $A_{X_1}^2$ in Eq. (53) is in proportion to the parameter λ . Additionally, in this parameter space, $m_H \approx m_A \approx 2.2$ TeV, $m_{S_R} \approx 1.5$ TeV, and $m_{P_R} \approx 1.1$ TeV, for $\tan\beta = 6$ and $\lambda = 0.1$. Therefore, for $m_A \sim \mathcal{O}(\text{TeV})$, we can believe that the parameter space is in the large m_A limit, and accordingly the approximate expressions Eq. (34) and Eq. (57) will work well. Meanwhile $m_{S_R} \sim \mathcal{O}(\text{TeV})$, and the approximate expression Eq. (58) is also consistent with the exact one.

We also picture the lightest Higgs boson mass m_h varying with the parameter κ in Fig. 2(a), where the solid line and dash-dot line denote m_h and m_{H_1} as $\tan\beta = 20$, the dash line and dot line denote m_h and m_{H_1} as $\tan\beta = 6$. And the quantity ξ_h varies with

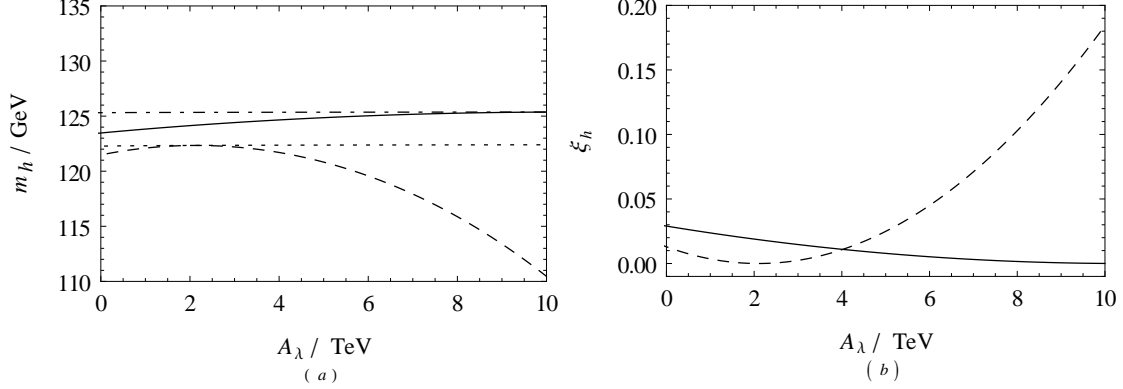


FIG. 3: (a) m_h varies with A_λ , the solid line and dash-dot line denote m_h and m_{H_1} as $\tan \beta = 20$, the dash line and dot line denote m_h and m_{H_1} as $\tan \beta = 6$. (b) ξ_h varies with A_λ , the solid line and dash line represent as $\tan \beta = 20$ and $\tan \beta = 6$, respectively. When $\kappa = 0.4$, $\lambda = 0.1$ and $v_{\nu^c} = 2$ TeV.

the parameter κ in Fig. 2(b), where the solid line and dash line represent as $\tan \beta = 20$ and $\tan \beta = 6$, respectively. Here, we take $\lambda = 0.1$, $A_\lambda = 500$ GeV and $v_{\nu^c} = 2$ TeV. We can see that the lightest Higgs boson mass m_h deviates from the mass m_{H_1} largely, when the parameter κ is small. Of course, for small κ , the quantity ξ_h is large. Constrained by the Landau pole condition [5], we choose the parameter $\kappa \leq 0.6$.

In Fig. 3(a), for $\kappa = 0.4$, $\lambda = 0.1$ and $v_{\nu^c} = 2$ TeV, we draw the lightest Higgs boson mass m_h , varying with the parameter A_λ , where the solid line and dash-dot line denote m_h and m_{H_1} as $\tan \beta = 20$, the dash line and dot line denote m_h and m_{H_1} as $\tan \beta = 6$. And Fig. 3(b) shows the quantity ξ_h versus A_λ , where the solid line and dash line represent as $\tan \beta = 20$ and $\tan \beta = 6$, respectively. The numerical results show that $m_h \simeq m_{H_1}$ and $\xi_h \simeq 0$ as $A_\lambda \approx 2$ TeV for $\tan \beta = 6$, and as $A_\lambda \approx 10$ TeV for $\tan \beta = 20$, which is in accordance with Eq. (55). Comparing with the large tree-level contributions, the small one-loop contributions can be ignored, then Eq. (55) can be approximated as

$$A_\lambda \simeq 2v_{\nu^c} \left(\frac{3\lambda}{\sin 2\beta} - \kappa \right). \quad (63)$$

Therefore, when A_λ is around $2v_{\nu^c} \left(3\lambda/\sin 2\beta - \kappa \right)$, we could regard the lightest Higgs boson mass as $m_h \approx m_{H_1}$. If A_λ drifts off the value of $2v_{\nu^c} \left(3\lambda/\sin 2\beta - \kappa \right)$ significantly, the lightest Higgs boson mass m_h will deviate from the mass m_{H_1} .

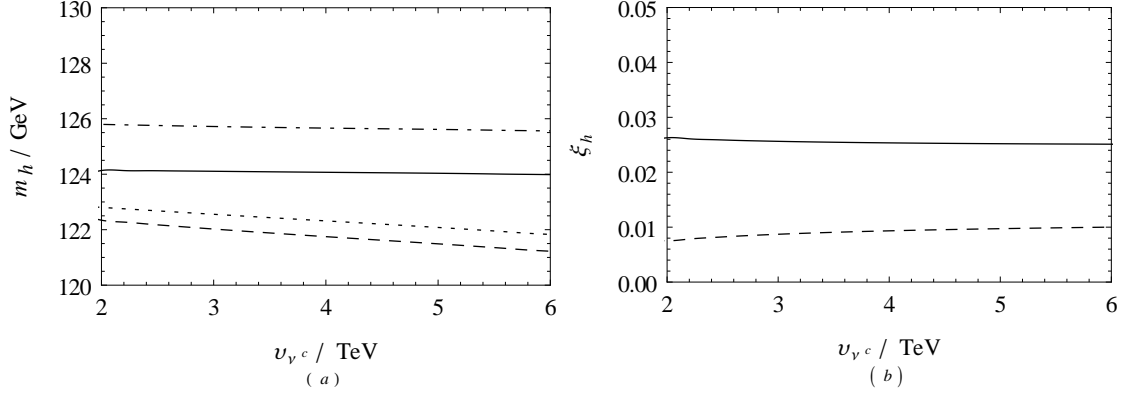


FIG. 4: (a) m_h varies with v_{ν^c} , the solid line and dash-dot line denote m_h and m_{H_1} as $\tan \beta = 20$, the dash line and dot line denote m_h and m_{H_1} as $\tan \beta = 6$. (b) ξ_h varies with v_{ν^c} , the solid line and dash line represent as $\tan \beta = 20$ and $\tan \beta = 6$, respectively. When $\kappa = 0.4$, $\lambda = 0.1$ and $A_\lambda = 500$ GeV.

Finally, for $\kappa = 0.4$, $\lambda = 0.1$, and $A_\lambda = 500$ GeV, we plot the lightest Higgs boson mass m_h versus the parameter v_{ν^c} in Fig. 4(a), where the solid line and dash-dot line denote m_h and m_{H_1} as $\tan \beta = 20$, and the dash line and dot line denote m_h and m_{H_1} as $\tan \beta = 6$. Fig. 4(b) shows ξ_h varying with v_{ν^c} , where the solid line and dash line represent as $\tan \beta = 20$ and $\tan \beta = 6$, respectively. We can see that the lightest Higgs boson mass m_h is parallel to the mass m_{H_1} with increasing of v_{ν^c} . Through Eq. (41), $m_{R_1}^2 \sim \mathcal{O}(v_{\nu^c}^2)$, and $A_{X_1}^2 \sim \mathcal{O}(v_{\nu^c})$ as shown in Eq. (53). Therefore, the quantity $\xi_h = \frac{(A_{X_1}^2)^2}{m_{R_1}^2 m_{H_1}^2}$ defined in Eq. (59) becomes flat with an increase of v_{ν^c} , which can be seen in Fig. 4(b). In addition, Fig. 4(a) indicates that m_h and m_{H_1} are decreasing slowly, with an increase of v_{ν^c} , because the parameter $\mu = 3\lambda v_{\nu^c}$, which can affect the radiative corrections for the lightest Higgs boson mass.

V. SUMMARY

In the framework of the $\mu\nu$ SSM, the three singlet right-handed neutrino superfields $\hat{\nu}_i^c$ are introduced to solve the μ problem of the MSSM. Correspondingly, the right-handed sneutrino VEVs lead to the mixing of the neutral components of the Higgs doublets with the sneutrinos, which produce a large CP-even neutral scalar mass matrix. Therefore, the mixing would affect the lightest Higgs boson mass. In this work, we consider the Higgs

boson mass radiative corrections with effective potential methods and then analytically diagonalize the CP-even neutral scalar mass matrix. Meanwhile, in the large m_A limit, we give an approximate expression for the lightest Higgs boson mass seen in Eq. (57). In numerical analysis, we analyze how the key parameters λ , $\tan\beta$, κ , A_λ , and v_{ν^c} affect the lightest Higgs boson mass.

Acknowledgments

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Appendix A: The masses for the third fermions and their superpartners

The masses for the third fermions $f = t, b, \tau$ are

$$m_t = Y_t |H_u^0|, \quad m_b = Y_b |H_d^0|, \quad m_\tau = Y_\tau |H_d^0|. \quad (\text{A1})$$

The corresponding 2×2 $\tilde{f}_L - \tilde{f}_R$ ($\tilde{f} = \tilde{t}, \tilde{b}, \tilde{\tau}$) mass squared matrices are

$$M_{\tilde{f}}^2 = \begin{pmatrix} M_{\tilde{f}_L}^2 & M_{X_f}^2 \\ M_{X_f}^{2*} & M_{\tilde{f}_R}^2 \end{pmatrix}, \quad (\tilde{f} = \tilde{t}, \tilde{b}, \tilde{\tau}) \quad (\text{A2})$$

where the concrete expressions for matrix elements can be given as

$$M_{\tilde{t}_L}^2 = m_{\tilde{Q}_3}^2 + \frac{3g_2^2 - g_1^2}{12} (|H_d^0|^2 - |H_u^0|^2) + Y_t^2 |H_u^0|^2, \quad (\text{A3})$$

$$M_{\tilde{t}_R}^2 = m_{\tilde{U}_3}^2 + \frac{g_1^2}{3} (|H_d^0|^2 - |H_u^0|^2) + Y_t^2 |H_u^0|^2, \quad (\text{A4})$$

$$M_{X_t}^2 = Y_t (A_t |H_u^0| - \lambda_t \tilde{\nu}_i^{c*} H_d^0), \quad (\text{A5})$$

$$M_{\tilde{b}_L}^2 = m_{\tilde{Q}_3}^2 - \frac{3g_2^2 + g_1^2}{12} (|H_d^0|^2 - |H_u^0|^2) + Y_b^2 |H_d^0|^2, \quad (\text{A6})$$

$$M_{\tilde{b}_R}^2 = m_{\tilde{D}_3}^2 - \frac{g_1^2}{6}(|H_d^0|^2 - |H_u^0|^2) + Y_b^2 |H_d^0|^2, \quad (\text{A7})$$

$$M_{X_b}^2 = Y_b(A_b H_d^0 - \lambda_i \tilde{\nu}_i^{c*} H_u^{0*}), \quad (\text{A8})$$

$$M_{\tilde{\tau}_L}^2 = m_{\tilde{L}_3}^2 + \frac{g_1^2 - g_2^2}{4}(|H_d^0|^2 - |H_u^0|^2) + Y_\tau^2 |H_d^0|^2, \quad (\text{A9})$$

$$M_{\tilde{\tau}_R}^2 = m_{\tilde{E}_3}^2 - \frac{g_1^2}{2}(|H_d^0|^2 - |H_u^0|^2) + Y_\tau^2 |H_d^0|^2, \quad (\text{A10})$$

$$M_{X_\tau}^2 = Y_\tau(A_\tau H_d^0 - \lambda_i \tilde{\nu}_i^{c*} H_u^{0*}). \quad (\text{A11})$$

Here we ignore the small terms including Y_ν or $|\tilde{\nu}_i|$. The eigenvalues $m_{\tilde{f}_{1,2}}^2$ of the $\tilde{f} = \tilde{t}, \tilde{b}, \tilde{\tau}$ mass squared matrices can be given by

$$m_{\tilde{f}_{1,2}}^2 = \frac{M_{\tilde{f}_L}^2 + M_{\tilde{f}_R}^2}{2} \pm \sqrt{\left(\frac{M_{\tilde{f}_L}^2 - M_{\tilde{f}_R}^2}{2}\right)^2 + |M_{X_f}^2|^2}. \quad (\text{A12})$$

If substituting the VEVs for the corresponding neutral scalars, the masses of the third fermions $f = t, b, \tau$ and their superpartners are manifestly obtained.

Appendix B: The corrections to the minimization conditions

Considering one-loop corrections to the minimization conditions from the third fermions $f = t, b, \tau$ and their superpartners, ΔT_{H_d} , ΔT_{H_u} , and $\Delta T_{\tilde{\nu}_{ij}^c \nu_{\nu_j}^c}$ are given below:

$$\begin{aligned} \Delta T_{H_d} = & \frac{3}{(4\pi)^2} \left\{ \frac{G^2}{8} [f(m_{\tilde{t}_1}^2) + f(m_{\tilde{t}_2}^2)] - [Y_t^2 \mu (A_t \tan \beta - \mu) \right. \\ & - \frac{3g_2^2 - 5g_1^2}{24} (m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2)] \frac{f(m_{\tilde{t}_1}^2) - f(m_{\tilde{t}_2}^2)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \\ & + \left(Y_b^2 - \frac{G^2}{8} \right) [f(m_{\tilde{b}_1}^2) + f(m_{\tilde{b}_2}^2)] - 2Y_b^2 f(m_b^2) \\ & + \left[Y_b^2 A_b (A_b - \mu \tan \beta) - \frac{3g_2^2 - g_1^2}{24} (m_{\tilde{b}_L}^2 - m_{\tilde{b}_R}^2) \right] \frac{f(m_{\tilde{b}_1}^2) - f(m_{\tilde{b}_2}^2)}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} \Big\} \\ & + \frac{1}{(4\pi)^2} \left\{ \left(Y_\tau^2 - \frac{G^2}{8} \right) [f(m_{\tilde{\tau}_1}^2) + f(m_{\tilde{\tau}_2}^2)] - 2Y_\tau^2 f(m_\tau^2) \right. \\ & + \left[Y_\tau^2 A_\tau (A_\tau - \mu \tan \beta) - \frac{g_2^2 - 3g_1^2}{8} (m_{\tilde{\tau}_L}^2 - m_{\tilde{\tau}_R}^2) \right] \frac{f(m_{\tilde{\tau}_1}^2) - f(m_{\tilde{\tau}_2}^2)}{m_{\tilde{\tau}_1}^2 - m_{\tilde{\tau}_2}^2} \Big\}, \quad (\text{B1}) \\ \Delta T_{H_u} = & \frac{3}{(4\pi)^2} \left\{ \left(Y_t^2 - \frac{G^2}{8} \right) [f(m_{\tilde{t}_1}^2) + f(m_{\tilde{t}_2}^2)] - 2Y_t^2 f(m_t^2) \right. \end{aligned}$$

$$\begin{aligned}
& + \left[Y_t^2 A_t (A_t - \mu \cot \beta) - \frac{3g_2^2 - 5g_1^2}{24} (m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2) \right] \frac{f(m_{\tilde{t}_1}^2) - f(m_{\tilde{t}_2}^2)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \\
& + \frac{G^2}{8} \left[f(m_{\tilde{b}_1}^2) + f(m_{\tilde{b}_2}^2) \right] - \left[Y_b^2 \mu (A_b \cot \beta - \mu) \right. \\
& \left. - \frac{3g_2^2 - g_1^2}{24} (m_{\tilde{b}_L}^2 - m_{\tilde{b}_R}^2) \right] \frac{f(m_{\tilde{b}_1}^2) - f(m_{\tilde{b}_2}^2)}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} \Big\} \\
& + \frac{1}{(4\pi)^2} \left\{ \frac{G^2}{8} \left[f(m_{\tilde{\tau}_1}^2) + f(m_{\tilde{\tau}_2}^2) \right] - \left[Y_\tau^2 \mu (A_\tau \cot \beta - \mu) \right. \right. \\
& \left. \left. - \frac{g_2^2 - 3g_1^2}{8} (m_{\tilde{\tau}_L}^2 - m_{\tilde{\tau}_R}^2) \right] \frac{f(m_{\tilde{\tau}_1}^2) - f(m_{\tilde{\tau}_2}^2)}{m_{\tilde{\tau}_1}^2 - m_{\tilde{\tau}_2}^2} \right\}, \tag{B2}
\end{aligned}$$

$$\begin{aligned}
\Delta T_{\tilde{\nu}_{ij}^c \nu_{\nu_j^c}} &= \frac{3}{(4\pi)^2} \left\{ \lambda_i Y_t^2 v_d^2 (\lambda_j \nu_{\nu_j^c} - A_t \tan \beta) \frac{f(m_{\tilde{t}_1}^2) - f(m_{\tilde{t}_2}^2)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \right. \\
& + \lambda_i Y_b^2 v_u^2 (\lambda_j \nu_{\nu_j^c} - A_b \cot \beta) \frac{f(m_{\tilde{b}_1}^2) - f(m_{\tilde{b}_2}^2)}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} \Big\} \\
& + \frac{1}{(4\pi)^2} \left\{ \lambda_i Y_\tau^2 v_u^2 (\lambda_j \nu_{\nu_j^c} - A_\tau \cot \beta) \frac{f(m_{\tilde{\tau}_1}^2) - f(m_{\tilde{\tau}_2}^2)}{m_{\tilde{\tau}_1}^2 - m_{\tilde{\tau}_2}^2} \right\}, \tag{B3}
\end{aligned}$$

with $\mu = \lambda_i \nu_{\nu_i^c}$, $f(m^2) = m^2 (\log \frac{m^2}{Q^2} - 1)$.

Appendix C: The diagonalization of the 3×3 mass matrix

The eigenvalues of the 3×3 mass squared matrix \mathcal{H} are given as [21, 56]

$$m_1^2 = \frac{a}{3} - \frac{1}{3}p(\cos \phi + \sqrt{3} \sin \phi), \tag{C1}$$

$$m_2^2 = \frac{a}{3} - \frac{1}{3}p(\cos \phi - \sqrt{3} \sin \phi), \tag{C2}$$

$$m_3^2 = \frac{a}{3} + \frac{2}{3}p \cos \phi. \tag{C3}$$

To formulate the expressions in a concise form, one can define the notations,

$$p = \sqrt{a^2 - 3b}, \tag{C4}$$

$$\phi = \frac{1}{3} \arccos \left(\frac{1}{p^3} (a^3 - \frac{9}{2}ab + \frac{27}{2}c) \right), \tag{C5}$$

with

$$a = \text{Tr}(\mathcal{H}), \tag{C6}$$

$$b = \mathcal{H}_{11}\mathcal{H}_{22} + \mathcal{H}_{11}\mathcal{H}_{33} + \mathcal{H}_{22}\mathcal{H}_{33} - \mathcal{H}_{12}^2 - \mathcal{H}_{13}^2 - \mathcal{H}_{23}^2, \quad (\text{C7})$$

$$c = \text{Det}(\mathcal{H}). \quad (\text{C8})$$

In a general way, $m_1^2 \leq m_2^2 \leq m_3^2$. So, one can have two possibilities on the mass spectrum:

(i) spectrum with $m_h < m_H \leq m_{S_3}$:

$$m_h^2 = m_1^2, \quad m_H^2 = m_2^2, \quad m_{S_3}^2 = m_3^2, \quad (\text{C9})$$

(ii) spectrum with $m_h < m_{S_3} < m_H$:

$$m_h^2 = m_1^2, \quad m_H^2 = m_3^2, \quad m_{S_3}^2 = m_2^2. \quad (\text{C10})$$

The normalized eigenvectors for the mass squared matrix \mathcal{H} are given by

$$\begin{pmatrix} U_{X_{11}} \\ U_{X_{21}} \\ U_{X_{31}} \end{pmatrix} = \frac{1}{\sqrt{|X_1|^2 + |Y_1|^2 + |Z_1|^2}} \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix}, \quad (\text{C11})$$

$$\begin{pmatrix} U_{X_{12}} \\ U_{X_{22}} \\ U_{X_{32}} \end{pmatrix} = \frac{1}{\sqrt{|X_2|^2 + |Y_2|^2 + |Z_2|^2}} \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix}, \quad (\text{C12})$$

$$\begin{pmatrix} U_{X_{13}} \\ U_{X_{23}} \\ U_{X_{33}} \end{pmatrix} = \frac{1}{\sqrt{|X_3|^2 + |Y_3|^2 + |Z_3|^2}} \begin{pmatrix} X_3 \\ Y_3 \\ Z_3 \end{pmatrix}, \quad (\text{C13})$$

with

$$X_1 = (\mathcal{H}_{22} - m_h^2)(\mathcal{H}_{33} - m_h^2) - \mathcal{H}_{23}^2, \quad (\text{C14})$$

$$Y_1 = \mathcal{H}_{13}\mathcal{H}_{23} - \mathcal{H}_{12}(\mathcal{H}_{33} - m_h^2), \quad (\text{C15})$$

$$Z_1 = \mathcal{H}_{12}\mathcal{H}_{23} - \mathcal{H}_{13}(\mathcal{H}_{22} - m_h^2), \quad (\text{C16})$$

$$X_2 = \mathcal{H}_{13}\mathcal{H}_{23} - \mathcal{H}_{12}(\mathcal{H}_{33} - m_H^2), \quad (\text{C17})$$

$$Y_2 = (\mathcal{H}_{11} - m_H^2)(\mathcal{H}_{33} - m_H^2) - \mathcal{H}_{13}^2, \quad (\text{C18})$$

$$Z_2 = \mathcal{H}_{12}\mathcal{H}_{13} - \mathcal{H}_{23}(\mathcal{H}_{11} - m_H^2), \quad (\text{C19})$$

$$X_3 = \mathcal{H}_{12}\mathcal{H}_{23} - \mathcal{H}_{13}(\mathcal{H}_{22} - m_{S_3}^2), \quad (\text{C20})$$

$$Y_3 = \mathcal{H}_{12}\mathcal{H}_{13} - \mathcal{H}_{23}(\mathcal{H}_{11} - m_{S_3}^2), \quad (\text{C21})$$

$$Z_3 = (\mathcal{H}_{11} - m_{S_3}^2)(\mathcal{H}_{22} - m_{S_3}^2) - \mathcal{H}_{12}^2. \quad (\text{C22})$$

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